Psychological Preference-based Optimization Framework with Interactive Genetic Algorithms on the Nurse Scheduling Problem

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Abstract
One of the challenges of the nurse scheduling problems (NSPs) is the great number of constraints, and another challenge is that psychological preference of chief nurse is usually difficult to clearly define. Typically, solutions to conquer the first challenge rely on operational research (OR) methods, and the objective functions are constructed manually to conquer the second challenge. However, OR methods are usually time consuming and it is too strong an assumption that the constructed objective function is close to the psychological preference in chief nurses’ minds. This paper presents a psychological preference-based optimization framework (PPOF) with a guidable hill-climbing decoder (HCD) and the active interactive genetic algorithm (AIGA). The guidable HCD yields satisfactory performance in speed, and it reduces the highly-constrained search space to a constraint-less one which is easy for genetic algorithms to optimize. To meet the psychological preference, PPOF adopts AIGA to tune HCD to approximate the chief nurses’ psychological preference. The experiment shows that PPOF is able to arrange a feasible monthly schedule of a realistic NSP of the National Taiwan University Hospital within several seconds.

1 Introduction
Nurse scheduling is an important issue faced by many if not all hospitals. The objective of nurse scheduling is to supply enough manpower needs of different skills, balance workload among nurses, follow the code of practice of the workplace, increase employee satisfaction of staff, and improve efficiency of daily routine. Due to different medical treatments, needs and constraints are generally different in different wards. Besides, some informal agreements are usually generated from nurses’ work experiences, and those agreements are usually not revealed from clauses of the code of practice. Due to these significant differences, schedules are usually arranged by chief nurses and often becomes a time consuming task.

The nurse scheduling problem (NSP) or nurse rostering is a well-known NP-hard combinatorial optimization problem in literature (Osogami & Imai, 2000). General overviews of NSP can be found in (Burke, Causmaecker, Berghe, & Landeghem, 2004; Hung, 1995). Early researches concentrated on mathematical programming model (Abernathy, Baloff, Hershey, & Wandel, 1973; Miller, Pierskalla, & Rath, 1976). Due to computational complexity concern, those approaches were restricted to problems which were too simple to be practical for most hospitals. To solve more realistic NSP in reasonable time, investigations have been conducted in other approaches, including constraint programming (Okada & Okada, 1988), expert system (Chen & Yeung, 1993), tabu search (Dowsland, 1998), simulated annealing (Brusco & Jacobs, 1993), electromagnetism meta-heuristic (Maenhout & Vanhoucke, 2007), and genetic algorithms (GAs) (Easton & Mansour, 1993).

In most existing researches, the objective functions of GAs were constructed based on researcher’s estimations from the literature and careful consultations with hospital staff. The common assumption of those researches was that the objective fitness functions constructed by researchers were close to the psychological preference in chief nurses’ minds; however, this assumption is usually too strong to be true.
The problem in this paper is arisen at the National Taiwan University Hospital (NTUH). In this problem, the number of nurses is around twenty and the scheduling period is one month. The purpose of this paper is to solve the realistic NSP in NTUH and optimize the schedules according to chief nurses’ preference. We propose a psychological preference-based optimization framework (PPOF) which integrates a hill-climbing decoder (HCD) and an active interactive genetic algorithm (AIGA) (Llorà, Sastry, Goldberg, Gupta, & Lakshmi, 2005). The proposed HCD arranges a monthly schedule in reasonable time and it reduces the highly-constrained search space of scheduling table to a constraint-less one of few guidable parameters. PPOF adopts AIGA to search the optimal solution in the search space reducing by HCD. With such the optimal solution, PPOF could approximate the inexplicable objective function of HCD to the chief nurses’ psychological preference.

The remainder of this paper is structured as follows. Section 2 presents the short introduction to constraint handling techniques on GAs, NSP, and IEC. Section 3 describes the problem definition in this paper. Section 4 provides details of the proposed hill-climbing decoder and combination of HCD and AIGA. In section 5, the details of experiment and results is presented. Section 6 presents conclusions of this study.

2 Background

This section gives brief introductions to constraint-handling techniques, researches of GAs’ applications on NSPs and IEC.

2.1 Constraint-handling techniques on GAs

In real-world problems, constraints almost exist due to limited resource and demand. Therefore, constraint-handling is an important issue in optimization. There have been many constraint-handling techniques in applications of GAs including penalty fitness function, decoder, repair, multi-objective approach, mapping etc.. Analyzations and discussions of previous researches can be found elsewhere (Coello-Coello, 2002).

2.2 Previous work of GAs on NSPs

Most NSPs are complex and highly-constrained. Some researchers even believed that nurse scheduling is more complex than the traveling salesman problem (Tien & Kamiyama, 1982). Dowsland and Thompson proposed a three-stage decomposition of NSP (Dowsland & Thompson, 2000). In their problem, one day was partitioned into three shifts: early, late, and long night shift. Early and late shifts were considered as day shifts in the first and the second stage of their decomposition. The first stage adopted the knapsack model to check whether nurses were enough for latter stages. If not, bank nurses were allocated to supply manpower needs. The second stage was most difficult, and its task was to arrange day and night shifts on the scheduling table. Tabu search (TS) was applied in this stage. The third stage adopted a netflow model to assign early and late shifts from the day shifts arranged on the second stage. The main objective of the third stage was to reduce the computational complexity in the second stage. Experiment showed that the three-stage decomposition was able to find good solutions in their problem. In addition to TS, different meta-heuristic methods could be applied in the second stage.

Aickelin and Dowsland introduced a special encoding scheme and some problem specific local heuristic decoders in an indirect genetic algorithms (Aickelin & Dowsland, 2004) framework. A chromosome was encoded as decoder-index string. Each gene presented the decoder applied in the corresponding nurse’s schedule. A similar framework is applied in (Li & Aickelin, 2003) by Li and Aickelin where rule-base approach was used to mimic a human expert in scheduling process. Human experts generated high quality schedules due to the ability to switch between rules, based on current state of the schedules. A bayesian optimization algorithm (BOA) (Pelikan, Goldberg, & Cantú-Paz, 1999) was applied to play this role. In both approaches, a carefully designed penalty fitness function and the three-stage decomposition were applied. Experiments showed that both approaches have the ability to search more feasible solutions than other approaches in short time (Li & Aickelin, 2003).
2.3 Interactive Evolutionary Computation

Interactive evolutionary computation (IEC) is a method that human and evolutionary computation (EC) cooperate to search the optimal solutions in the psychological search space of the human. IEC reduces implicit psychological space into parameter-features space which is solvable to computer (Takagi, 2001). There are wide of applications using IEC like graphics (McCormack), music composition (Moroni, Manzolli, Zuben, & Gudwin, 2000), industry design (Kim & Cho, 2000) etc.

3 Problem Definition

The problem described here is to create monthly schedules of the medical ward containing up to 20 nurses at the National Taiwan University Hospital. The schedule must satisfy constraints of the work contract, obey the law, support the requirement of daily operations in different class grade, meet the day-off requested by nurses as much as possible, and fulfill preferred scheduling arrangement of chief nurse. The features of preferred scheduling arrangement forms soft constraints in this problem and such constraints would be introduced later.

A three-shift working system is adopted in this problem where \( A, E, \) and \( N \) represents day shift, evening shift, and night shift respectively. Generally, a nurse works on one working shift consistently through one month. Five class grades of nurses are N0, N1, N2, N3, and N4 where a more senior nurse has a greater N level. The requirements of different class grades are divided into two categories, senior and junior. The definition of the two categories is different in different wards and even in different working shifts, e.g., in medical ward, senior nurses are those whose class grade is above N2 on A and above N1 on N. One special character in NSPs, also arisen in this problem, is that the senior nurses can cover the junior nurses’ work, but not allowed otherwise. The coverage characteristic makes the scheduling more complex (Dowsland, 1998). A nurse can request days off on his/her preferred days of the month before chief nurse arranges all schedule. To increase employee satisfaction, requested day-off should be met if allowed. Two categories of requested day-off exist. The first is priority requested day-off which must be met under any circumstances. A priority day-off usually is an annual leave, a sick leave, or a maternity leave in this problem. The second is normal requested day-off which is not necessary but better to be met.

For simplicity, we use \( A \) to represent all kinds of working shifts. In the following part of this section, \( A \) can be substituted to \( E \) or \( N \) to apply evening or night shift except HC-6. Hard constraints define the feasibility of each solution, therefore hard constraints cannot be violated. The generic hard constraints in this problem are as follows:

**HC-1** The number of days off in a month must at least \( F_{\min} \) days.

\[
\sum_j f_{ij} \geq F_{\min}, \quad \forall i = \{1, ..., n\},
\]

\[
f_{ij} \in \{0, 1\},
\]

where \( F_{\min} \) is minimum days off in a month protected by law, and \( f_{ij} \) is 1 if nurse \( i \) is off on day \( j \), 0 otherwise And \( n \) is number of nurses.

**HC-2** Consecutive working days cannot be longer than \( A_{\text{con}} \) days.

\[
\prod_{k=j}^{j+A_{\text{con}}} a_{ik} = 0, \quad \forall j = \{1, ..., D - A_{\text{con}}\},
\]

\[
a_{ik} \in \{0, 1\},
\]

where \( A_{\text{con}} \) is maximum consecutive working days, \( a_{ij} \) is 1 if nurse \( i \) is working on day \( j \), and \( D \) is number of days in the scheduling month.

**HC-3** The number of nurses is at least \( R_j \) in different working shift.

\[
\sum_i a_{ij} \geq R_j, \quad \forall j.
\]


(a) Soft constraint 1 with $A_{pr} = 5$.

(b) Soft constraint 2 with $F_{pr} = 2$.

(c) Soft constraint 3.

Figure 1: Soft constraints

**HC-4** The number of senior nurses is at least $RS_j$ according to different working shift.

$$\sum_i s_i a_{ij} \geq RS_{ij}, \; \forall j,$$

where $R_j$, $RS_j$ are the minimum requirement of total nurses and senior nurses. And $s_i$ is 1 if nurse $i$ is senior, 0 otherwise.

**HC-5** Priority requested day-off must be met.

$$f_{ij} = 1, \; \forall i, j \in \text{Prior requested day-off.}$$

**HC-6** Day shift must not be allocated to the next day of each night shift.

$$a_{ij} \cdot a_{i,j+1} = 0, \text{ if } a_{ij} \in \text{night and } a_{i,j+1} \in \text{day},$$

where the decision variable $f_{ij}$ is the complement of $a_{ij}$.

$$f_{ij} = \bar{a}_{ij}.$$ (7)

The constraints established by Labor Standard Law (HC-1, HC-6) is not allowed for adjustment. Other constraints (HC-2 - HC-4) established by principle or tradition of the wards are adjustable in different months and wards, but such constraints are usually consistent. The definition of priority requested day-off is depending on, and usually on, chief nurses. Therefore, HC-5 is also adjustable to chief nurses by hand.

Soft constraints are introduced in this problem as the features of preferred scheduling arrangement. Unlike hard constraints, soft constraints are not necessary but better to be satisfied to produce preferred schedules. The generic soft constraints of this problem are as follows:

**SC-1** Consecutive working days are longer, best in $A_{pr}$ days, where $A_{pr}$ is preferred consecutive working days. Figure [La] shows this constraint.

**SC-2** Continuous days off are best in $F_{pr}$ days, where $F_{pr}$ is preferred consecutive off days. Figure [Lb] shows the example.

**SC-3** One working day surrounded by days off on the previous and the next day is avoiding. Figure [Lc] shows this situation. In the following of this paper, $OAO$ is used to represent this situation.

**SC-4** Workload on official holidays is balanced.
SC-5 Normal requested day-off are met.

These constraints are established from careful consultants. Some constraints are adjustable (SC-1 and SC-2). Chief nurses in different wards would have distinct preference about these constraints. As the result, a consistent fitness function is hard to established among different wards. However, chief nurses view SC-5 as the most priority soft constraints in this problem.

4 Methodology: PPOF

Figure 2 shows the flow of PPOF. PPOF has three components. First, guidable fast search (GFS) can efficiently produce the outputs under highly-constrained situations. The speed performance of GFS should be acceptable for human in interactions with PPOF. Second, PPOF adopts EC to tune the GFS according to the surrogate fitness which is synthesized from human preference. To combat user fatigue, the third component of PPOF is surrogate fitness synthesizer (SFS). PPOF would ask the user about the preference of the outputs, and SFS would synthesize a surrogate fitness function by user’s evaluations. PPOF adopted the EC to search the optimal solution of the GFS according to the surrogate function. With such solution, GFS is able to produce preferred outputs to user in reasonable time.

This paper proposed a decoder based on hill-climbing and branch-and-bound concepts to play the role of GFS. AIGA is adopted to play the parts of the EC and SFS. This section gives a brief introduction to AIGA and describes the details of HCD and the integration of HCD and AIGA under PPOF.

4.1 Active Interactive Genetic Algorithm

One of the challenges of IECs is combating with user’s fatigue. Even a moderate size problem in ECs would cost thousands of function evaluations for solving and the result is not probable for human interaction. Active interactive genetic algorithm (Llorà, Sastry, Goldberg, Gupta, & Lakshmi, 2005) was proposed by Llorà in 2005. AIGA combines partial ordering concepts, notion of non-domination from multi-objective optimization (Goldberg, 1989), and ε-insensitive support vector regression (ε-SVR) to synthesize a surrogate fitness model based on user’s evaluation. Two properties of synthetic fitness are addressed: (1) fitness extrapolation: synthetic fitness should provide meaningful interferences beyond the current partial order provided by the users, and (2) order maintenance: synthetic fitness must maintain the order under tournament selection scheme.

A basic assumption to synthesize the surrogate model is that the partial order of user’s evaluations could be translated into a global numeric value. AIGA constructs tournament sets from solutions. In binary tournament selection, a user is asked to decide which of the two solutions in a tournament set is better, or both are equal. Therefore, the tournament ordering is constructed by a user’s preference. The tournament ordering guarantees that the partial order produced a connected graph $G$. Such graph $G = \langle V, E \rangle$ represents the solutions as vertex in $V$, and the
pair-wise comparison among individuals (greater, lesser, or equal) as edges in $E$. A global ordering is computed using a heuristic based on Pareto dominance concept of multi-objective optimization. Once the global ordering is computed, such ordering is applied to train surrogate model by $\varepsilon$-SVR. The compact GA (cGA) (Harik, Lobo, & Goldberg, 1999) optimizes a look-a-head candidate solution from the surrogate model. The empirical results showed the improvements with speed-up ranging from 3 to 7 times. The speed-up would combat human fatigue efficiently. The details of AIGA could be found in (Llorà, Sastry, Goldberg, Gupta, & Lakshmi, 2005).

4.2 Hill-climbing Decoder

To solve a realistic NSP in reasonable time, a heuristic decoder is proposed. From the description of Section 3, three important characteristics of this problem can be conducted, three-shift working system, monthly consistent working shift of a nurse, and same constrained formality in different working shifts. Due to the three characters, the scheduling procedure can separate into three sub-schedules of $A$, $E$, and $N$. In this way, complexity computation are reduced. Three sub-decoders are designed based on hill-climbing and branch-and-bound concepts. Therefore, the hill-climbing decoder is a hierarchical architecture composed of one main-decoder and three sub-decoders.

There are three operational stages in scheduling process. Pre-scheduling stage executes on main-decoder before hill-climbing scheduling process on sub-decoders. Requested day-off permission stage and hill-climbing scheduling stage execute on sub-decoders. In requested day-off permission stage, the sub-decoder resolves the potential conflict over requested day-off, where such conflicts would lead to an infeasible solution. Hill-climbing scheduling stage arranges the schedule with hill-climbing and branch-and-bound concepts. The schedule is a two-dimensional string matrix in decoders where each entry for nurse $i$

$$x_{ij} = \begin{cases} 
A & \text{if works for day shift on day } j, \\
E & \text{if works for evening shift on day } j, \\
N & \text{if works for night shift on day } j, \\
O & \text{if is off on day } j, \\
U & \text{if is not decided yet on scheduling,}
\end{cases} \quad (8)$$

where $U$ is called unassigned shift in this paper.

4.2.1 Pre-scheduling Stage

In the pre-scheduling stage, main-decoder has three operations to do. First, rough manpower check ensures enough nurses in next stages. If nurses provided by chief nurse are not sufficient, the scheduling process would terminate. By the word rough, we mean that this check would be accurate when no day off is requested in the scheduling month.

$$\frac{n \times (D - F_{\text{min}})}{D} \geq R_j. \quad (9)$$

Second, linkage adjustment is applied to satisfy HC-6. Main-decoder tries to arrange a day off on the first day of the scheduling month when a nurse is working on $A$ and $N$ on the last month. If a day off is allowed, the arrangement is finished. If not, main-decoder would search nurses working on $E$ and $N$ to work day shift temporarily so that an off could be arranged on the first day of the nurse under HC-6 situation. Main-decoder would search iteratively though days until there is no feasible solution or adjust linkage successfully. The third operation is administrative personnel scheduling. This operation arrange the corresponding schedule of administrative personnel and has not affect on the scheduling of nurses. After finishing the pre-scheduling stage, main-decoder would call three sub-decoders to finished the schedules of $A$, $E$, and $N$. In the following section, we would use day shift, $A$, for introduction. The same framework could be used to other working shift by replacing $A$ with $E$ or $N$.

4.2.2 Requested Day-off Permission Stage

An important component called validation check is putted in use on this and next stage. Validation check predicts whether a feasible schedule exists at the current state of schedule. By temporarily
Algorithm 1 Hill-climbing decoder

1. Read parameters.
2. Rough manpower check, terminate if false.
3. Linkage adjust, terminate if false.
4. Administrative personnel scheduling.
5. Release sub-decoders.
6. Requested day-off permission, terminate if false.
7. Find hard entries, go to step 10 if found.
8. Calculate $p_{ij}$.
9. Assign holiday or working in most confident entry.
10. If there is any unassigned entry, go to step 7.
11. Combine scheduling table from sub-decoders with tables of administrative personnel.

assigning $A$ or $O$ in an entry, the sub-decoder could find out entries that must work or take off in order to find a feasible solution with validation check. Such entries are called hard entries.

This stage is designed to satisfy the constraints of requested day-off (HC-5, SC-5), where SC-5 is most prior to the other soft constraints in chief nurses’ opinions. The decoder checks both prior and normal requested day-off. If any priority requested day-off is invalid, the scheduling process would terminate due to conflict over hard constraints. If any normal requested day-off is invalid, the sub-decoder cancels the normal requested day-off to ensure feasible solution.

4.2.3 Hill-climbing Scheduling Stage

In this stage, the sub-decoder introduced two additional probability matrices which guide the trend of hill-climbing scheduling process iteratively. The minimum probability matrix $p_{min,ij}$ represents minimum probability to take off for nurse $i$ on day $j$, and maximum probability matrix $p_{max,ij}$ represents the maximum probability to take off. In every iteration, the decoder checks whether hard entries exist at first. If yes, the sub-decoder assigns working or off on hard entries and start next iteration. If not, the decoder calculates $p_{min,ij}$ and $p_{max,ij}$ according to HC-1, HC-3 and HC-4.

$$p_{min,ij} = \max\{\frac{F_{min} - \sum_j f_{ij}}{D_{U,i}}, 0\}, \forall i, j. \quad (10)$$

If nurse $i$ is junior, $p_{max,ij}$ is as follows

$$p_{max,ij} = \max\{0, \frac{n - R_j - \sum_i f_{ij}}{n_{U,j}}\}, \forall i, j. \quad (11)$$

If nurse $i$ is senior, $p_{max,ij}$ should be as follows

$$p_{max,ij} = \max\{0, \min\{\frac{n - R_j - \sum_i f_{ij}}{n_{U,j}}, \frac{ns - RS_j - \sum_i s_if_{ij}}{n_{U,j}}\}\}, \forall i, j. \quad (12)$$

Let parameters $D_{U,i}$ be number of days still unassigned for nurse $i$, $n_{U,j}$ be number of nurses still unassigned on day $j$, and $ns$ be number of senior nurses.

To get the final probability, the sub-decoder needs to calculate following four parameters. $C_{cw,ij}$ is the contribution of SC-1 for nurse $i$ on day $j$, where for nurse $i$

$$C_{cw,ij} = \begin{cases} \frac{\Delta_c}{t} & \text{if works for } A_{pr} - t \text{ consecutive days}, \\ 0 & \text{if neighbor entries are } U \text{ or } O, \\ -\Delta_cw & \text{if consecutive working days } > A_{pr}, \end{cases} \quad (13)$$

7
where $t > 0$. The contribution of SC-2, $C_{co,ij}$ is

$$
C_{co,ij} = \begin{cases} 
\Delta_{co} & \text{if works for } F_{pr} - t \text{ consecutive days}, \\
0 & \text{if neighbor entries are } U \text{ or } O, \\
-\Delta_{co} & \text{if consecutive working days } > F_{pr}.
\end{cases}
$$

The $C_{OAO,ij}$ is

$$
C_{OAO,ij} = \begin{cases} 
\Delta_{OAO} & \text{if neighbor entries are } O, \\
-\Delta_{OAO} & \text{if previous entries are } OA \text{ or next entries are } AO, \\
0 & \text{else}.
\end{cases}
$$

The contribution $C_{hb,ij}$ is as follows,

$$
F_{avg,oh} = \frac{\sum_{j \in \text{OH}} (n - R_j)}{n},
$$

$$
C_{hb,ij} = \begin{cases} 
\Delta_{hb} & \text{if } \sum_{j \in \text{OH}} f_{ij} < F_{avg,oh}, \\
0 & \text{otherwise},
\end{cases}
$$

where parameters $F_{avg,oh}$ is the average days off on official holidays should be taken for single nurse, and $\Delta_{cw}$, $\Delta_{co}$, $\Delta_{OAO}$, $\Delta_{hb}$ are decision variable which could be adjust by user to meet user’s preference. Finally, decoder calculates the probability

$$
p_{ij} = \max\{0, \min\{100, \frac{1}{2}p_{max,ij} + \frac{1}{2}p_{min,ij} + C_{cw,ij} + C_{co,ij} + C_{OAO,ij} + C_{hb,ij}\}\}.
$$

The distance from $p_{ij}$ to 100 represents the confidence to take off on this entry, and distance to 0 represents working likewise. The more smaller the distance, the more certain result decoder will confirms. As the result, decoder chooses most confident entry and assigns off if $p_{ij}$ is closer to 100 or working if $p_{ij}$ is closer to 0. If there is any unassigned shift in scheduling table, sub-decoder would start next iteration.

### 4.2.4 Putting It All Together

HCD provides as a fast scheduling method in a realistic NSP and a reducing from a highly-constrained search space on scheduling table to an unconstrained search space on four parameters, $\Delta_{cw}$, $\Delta_{co}$, $\Delta_{OAO}$, and $\Delta_{hb}$. The inexplicable objective function of decoder could match to human’s psychological preference by tuning these parameters. HCD would always arrange a feasible schedule whenever there is feasible solutions from the input data, i.e., requested day-off nurses, and constraints. This HCD provides a flexible architecture under three-shift working system. The steps of HCD are described in Algorithm 1.

### 4.3 HCD + AIGA

To arrange a preferred schedule, PPOF adopts AIGA to tune the inexplicable objective function of HCD to approximate the psychological preference of chief nurses. Figure 3 shows the block diagram of the PPOF in this problem.

The original flow of AIGA needs some modifications to be adopted in PPOF. First, AIGA converts binary string to real-value parameters. HCD arranges a feasible schedule with such parameters. A binary encoding scheme is chose to represent each parameter in HCD with 16 bits. Therefore, the length of chromosome was: 4 parameters $\times$ 16 bits = 64 bits. The value of parameters is restricted in the interval $[0,30]$.

In order to generate approximate synthetic fitness function, the solutions in tournament set should be diverse to explore more search space. Therefore, the second modification is that the solutions in $S'$ set are $2^{h-2}$ solutions sampling out of the probabilistic model evolved by cGA. The others $2^{h-2}$ solutions generated randomly where $h \geq 2$. Algorithm 2 describes the detail of PPOF in this problem.
Figure 3: Block diagram of HCD+AIGA, where $g(\Delta_{cw}, \Delta_{co}, \Delta_{DAO}, \Delta_{hb})$ is the inexplicable objective function of HCD, and $f_s(\Delta_{cw}, \Delta_{co}, \Delta_{DAO}, \Delta_{hb})$ is the surrogate function.

Algorithm 2 PPOF: HCD + AIGA

1. Create an empty directed graph $G$.
2. Create $2^h$ random initial solutions ($S$ set).
3. Create the hierarchical tournament set $T$ using the available solutions in $S$.
4. Convert binary string to parameters in $T$, and scheduling with hill-climbing decoder.
5. Present the tournaments in $T$ to the agent and update the partial ordering in $G$.
6. Estimate estimated ranking for each $v \in S$.
7. Train the surrogate $\varepsilon$-SVR surrogate synthetic fitness based on $S$ and estimated ranking.
8. Optimize the synthetic fitness using the cGA.
9. Create a $S'$ set, where $S \cap S' = \emptyset$, with $2^{h-2}$ new different solutions sampling out of the probabilistic mode evolved by cGA and $2^{h-2}$ new different solutions generated randomly.
10. Create hierarchical tournament set $T'$ with $2^{h-1}$ tournaments using $2^{h-1}$ solutions in $S$ and $2^{h-1}$ solutions in $S'$.
11. $S \leftarrow S \cup S'$.
12. $T \leftarrow T \cup T'$.
13. Go to step 4 till converged.

5 Experiment

To test the performance of PPOF, an agent is adopted to play the role of the chief nurse. PPOF would asks the agent which of the two schedules is preferred.

5.1 Agent

The proposed optimization system would ask an agent which of the two schedules is preferred or both are equal. An agent must has the ability to evaluate a monthly schedule. The evaluation function of the agent is designed according to the five soft constraints. Because the SC-5 is most priority, HCD only cancels the normal requested day-off which lead to infeasible schedule at requested day-off permission stage. The satisfaction at SC-5 would be same on the problem. Therefore, the SC-5 would not designed into the evaluation function. As mentioned before, HCD would arrange a feasible schedules whenever there is feasible solutions in the problem. The feasibility would not be designed into the evaluation function neither.
Table 1: Data of medical ward of the NTUH on January, 2010

<table>
<thead>
<tr>
<th>Common parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of nurses (A/E/N)</td>
<td>12/4/5</td>
</tr>
<tr>
<td>Scheduling period</td>
<td>31</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Hard constraints</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum consecutive working days (A/E/N)</td>
<td>6/5/4</td>
</tr>
<tr>
<td>Minimum requirement of nurses on weekdays (A/E/N)</td>
<td>5/3/2</td>
</tr>
<tr>
<td>Minimum requirement of senior nurses on weekdays (A/E/N)</td>
<td>1/1/1</td>
</tr>
<tr>
<td>Minimum requirement of nurses on holidays (A/E/N)</td>
<td>4/3/3</td>
</tr>
<tr>
<td>Minimum requirement of senior nurses on holidays (A/E/N)</td>
<td>1/1/1</td>
</tr>
<tr>
<td>Minimum off days in this month</td>
<td>6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Soft constraints</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Preferred consecutive workings days</td>
<td>4</td>
</tr>
<tr>
<td>Preferred consecutive off days</td>
<td>2</td>
</tr>
</tbody>
</table>

\[
f_{cw} = \frac{D_{cw}}{D_w},
\]

\[
f_{co} = \frac{D_{co}}{D_o},
\]

\[
f_{OAO} = \frac{D_w - D_{OAO}}{D_w},
\]

where \( f_{cw}, f_{co}, \) and \( f_{OAO} \) are the fitness values of SC-1, SC-2, and SC-3, respectively. \( D_{cw} \) is the number of days working for preferred consecutive working days in schedule, \( D_{co} \) is number of off days for preferred consecutive off days in schedule, \( D_{OAO} \) is number of days which one working day is surrounded by two off days in schedule, \( D_w \) is number of working days in schedule, and \( D_o \) is number of off days in schedule. The fitness value of SC-4 is conducted by number of nurses whose off days on official holidays is fewer than the average:

\[
f_{hb} = \frac{n}{\sum_{i} n_{ubh,i}},
\]

\[
n_{ubh,i} = \begin{cases} 1 & \text{if } |d_{fh,i} - F_{avg,oh}| > 1.0 \\ 0 & \text{otherwise} \end{cases},
\]

where \( n \) is number of nurses, \( d_{fh,i} \) is number of off days rest on official holidays, and \( n_{ubh} \) is number of nurses with unbalanced workload on official holidays. If nurse \( i \) is junior, \( F_{avg,oh} \) can be derived:

\[
F_{avg,oh} = \frac{(n - R) \times d_h}{n},
\]

where \( R \) is number of required nurses, and \( d_h \) is number of official holidays. If nurse \( i \) is senior instead:

\[
F_{avg,oh} = \min\left(\frac{n_s - RS_j}{n_s}, \frac{(n - R) \times d_h}{n}\right),
\]

where \( RS_j \) is number of required senior nurses, and \( n_s \) is number of senior nurses. The final fitness is combination of the above four fitness:

\[
F = w_{cw} \times f_{cw} + w_{co} \times f_{co} + w_{OAO} \times f_{OAO} + w_{hb} \times f_{hb},
\]

\[
w_{cw} + w_{co} + w_{OAO} + w_{hb} = 1.0,
\]

where \( w_{cw}, w_{co}, w_{OAO}, \) and \( w_{hb} \) are weights of four soft constraints. The agent is able to simulate different kind of chief nurses through adjusting the weights.
5.2 Experimental Results

The data of the medical ward on January, 2010 are given by NTUH. The data consist of parameters of both hard and soft constraints, requested day-off and monthly information like special official holidays in January, 2010. The parameters of this realistic NSP data are list in Table 1.

The agent simulates two types of chief nurses. Agent A prefers consecutive working and off. Agent B is extremely uncomfortable to find OAO in schedule. Two sets of weights $(w_{cw}, w_{co}, w_{OAO}, w_{hb}) = (0.35, 0.35, 0.15, 0.15)$ and $(0.15, 0.15, 0.55, 0.15)$. The chromosome length is 64 and the max number of consultations to agents is 90. Tournament size is $2^2 = 4$, where $h = 2$.

Figures 4 and 5 show the optimization processes for agents A and B. In these figures, the $x$-axis represents the number of consultants with the agent, and the $y$-axis represents the value of evaluation fitness of the agent. The experiment were constructed from one thousand independent runs. Moreover, the effect of the surrogate function synthesizer is investigated. Without the surrogate function synthesizer, cGA would evolve with user’s evaluation of schedules and the system would becomes HCD+cGA. The needed consultants of HCD+cGA is expected more than HCD+AIGA. The results show that the average evaluation fitness of both agents grows when number of consultants grows under HCD+AIGA. The results also show that the performance with the SFS is better in both agents. Such results indicate that the proposed PPOF with HCD+AIGA is able to meet psychological preference and the SFS has great influence on the optimization process.

The speed performance is listed in Table 2. The experiments were conducted on system with Q6600@2.4GHz and 2G ram. The BOA approach is proposed in (Li & Aickelin, 2003) and the approach is briefly introduced in Section 2.2. The constraint programming approach is testing on ilog OPL version 6.3 with CP engine. The parameters $(\Delta_{cw}, \Delta_{co}, \Delta_{OAO}, \Delta_{hb})$ used in HCD is (10.0, 10.0, 10.0, 10.0). Table 2 compares the result of the BOA approach which is unable to arrange a feasible schedule in this problem and the constraint programming approach which cost 26 seconds to arrange a feasible schedule. The proposed HCD is able to arrange a feasible schedule.
with given parameters. Such result implies that once the optimal set is given, HCD would reduce the time cost of chief nurses remarkably.

6 Conclusions

This paper proposed a psychological preference-based optimization framework which was an integration of constraint-handling techniques and IECs to solve NSP in the medical ward of NTUH and optimize the schedule according to chief nurses’ psychological preference. Specifically, an integration of HCD and AIGA was presented. The proposed stage-wise HCD had the ability to construct a complete monthly schedule in short time, usually within several seconds, which is acceptable for chief nurses. Furthermore, HCD reduced the highly-constrained search space to a constraint-less one which is easy for GAs to optimize. PPOF adopted AIGA to search the optimal parameters of HCD to approximate the inexplicable objective function to the chief nurses’ preference. Moreover, AIGA provided a surrogate function synthesizer based on partial ordering, multi-objective optimization ideas and ε-SVR. With the surrogate function, AIGA was able to reduce the number of evaluations required on the user.

The result showed that the proposed PPOF has the ability to meet psychological preference of different chief nurses. The proposed framework could optimize in shorter time than HCD+cGA framework which has no surrogate function synthesizer. The speed performance showed that HCD could arrange a feasible schedule in reasonable time. Such results indicated that once the optimal parameters set was evolved from the proposed framework. The chief nurse could schedule with HCD and the optimal set until his/her preference changes. Therefore, time cost of monthly scheduling time could be reduced.

There is still plenty of room for the psychological fitness evaluation in highly-constrained problem. For example, the effect of reducing search space of constraint-handling techniques is still unpredictable. The noise of judgments of human is not discussed in this paper. Also, instead of random sampling other techniques can be adopted to sample the search space like adaptive sampling, design of experiment, or discrete choice analysis.

We conclude some important characteristics in this PPOF.

1. The HCD has the ability to solve NSP in short time because longer waiting time for users might lead to incorrect judgments in evolutionary process.
2. The HCD reduces from a search space which is hard for ECs to easy-handling search space for ECs.
3. The AIGA framework can synthesize the psychological fitness function of human in short time to prevent human fatigue and variation of user’s criteria.
4. The AIGA framework is able to approximate the hidden objective function of the constraint-handling techniques to the synthetic psychological fitness function.

Although this paper focuses on NSP, the indications of the results are not limited to this specific type of problem. We believe that PPOF could be applied to many other problems with human interactions, highly-constrained and complicated computational context, but further investigation are required.

References


Table 2: Speed performance of different approaches.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Runtime</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hill-climbing Decoder</td>
<td>2 s</td>
</tr>
<tr>
<td>Constraint programming</td>
<td>26 s</td>
</tr>
<tr>
<td>BOA</td>
<td>N/A</td>
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</tbody>
</table>
693–711.


McCormack, J. Interactive evolution of l-system grammars for computer graphics modelling.


